

# Vibration Control of Tapering E-FGM Porous Wind Turbine Blades Using Piezoelectric Materials

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Active vibration control,  
piezoelectricity, tapered beam,  
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## ABSTRACT

Piezoelectric materials have many interesting applications thanks to their ability to generate an electrical voltage when subjected to mechanical pressure, and vice versa. In the field of vibration energy recovery, these materials can convert mechanical vibrations into electrical energy. For example, piezoelectric sensors integrated into roads or pavements can generate electricity from vibrations caused by vehicles or pedestrians. In the field of wind turbines, the vibratory control of wind turbine blades can increase the overall efficiency of these systems.

Additionally, composite materials are being increasingly integrated into the construction of mechanical systems, and specifically FGM materials. This work focuses on the active vibration control of FGM beams with non-uniform cross-sections, using piezoelectric materials. Euler-Bernoulli beam theory combined with the FEM is applied to an FGM beam. The equation of motion is generated using Hamilton's principle.

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## التحكم بالاهتزاز لشفرات توربينات الرياح المتدرجة وظيفياً ذات المسام والتقويس الآسي باستخدام المواد الكهروضغطية

خالد الحارثي ، محمد الطويل ، رشيد سعدني ، ميلود رحمون .

**ملخص:** تتمتع المواد الكهروضغطية بالعديد من التطبيقات المثيرة للاهتمام بفضل قدرتها على توليد جهد كهربائي عند تعرضها لضغط ميكانيكي، والعكس صحيح. في مجال استعادة طاقة الاهتزاز، يمكن لهذه المواد تحويل الاهتزازات الميكانيكية إلى طاقة كهربائية. على سبيل المثال، يمكن لأجهزة الاستشعار الكهروضغطية المدمجة في الطرق أو الأرصفة توليد الكهرباء من الاهتزازات التي تسببها المركبات أو المشاة. في مجال توربينات الرياح، يمكن للتحكم الاهتزازي في شفرات توربينات الرياح زيادة الكفاءة الإجمالية لهذه الأنظمة. بالإضافة إلى ذلك، يتم دمج المواد المركبة بشكل متزايد في بناء الأنظمة الميكانيكية، وخاصة المواد المتدرجة وظيفياً. يركز هذا العمل على التحكم النشط في الاهتزاز للعوارض المتدرجة وظيفياً ذات المقاطع العرضية غير المنتظمة، باستخدام المواد الكهروضغطية. يتم تطبيق نظرية العوارض لأويلر-برنولي جنباً إلى جنب مع طريقة العناصر المحدودة على عارضة متدرجة وظيفياً. يتم إنشاء معادلة الحركة باستخدام مبدأ هاملتون..

**الكلمات المفتاحية** – التحكم النشط في الاهتزاز، الكهرباء الانضغاطية، العارضة المدببة، المواد المتدرجة وظيفياً ذات المسام، شفرة التوربين.

### 1. INTRODUCTION

Piezoelectric materials are remarkable for their ability to convert mechanical pressure into electrical voltage and vice versa, a property that opens up a wide range of applications [1]. This unique characteristic stems from the crystalline structure of these materials, which facilitates the transformation of mechanical energy into electrical energy [2]. This phenomenon, known as the piezoelectric effect, is harnessed in various fields to create innovative solutions. Materials such as Lead Zirconate Titanate (PZT) ceramics and polyvinylidene fluoride (PVDF) polymers are among the most widely used piezoelectric materials. PZT is widely used due to its high piezoelectric constants and stability, making it suitable for applications in actuators, sensors, and transducers. PVDF, on the other hand, is valued for its flexibility and lightweight properties, which make it ideal for use in wearable technology and flexible electronics. In the field of vibration energy harvesting, piezoelectric materials play a crucial role. They have the ability to recover mechanical vibrations and transform them into electrical energy, which can then be used to power small electronic devices. This capability is particularly advantageous for portable devices, where battery life is a critical concern. Additionally, piezoelectric materials are employed in infrastructure monitoring systems to detect structural health and integrity. By converting vibrations from bridges, buildings, and other structures into electrical signals, these materials help in early detection of potential issues, thereby enhancing safety and maintenance efficiency [3]. Furthermore, piezoelectric materials are integral to the operation of wireless sensors. These sensors, which are often deployed in remote or hard-to-reach areas, rely on the energy harvested from ambient vibrations to function without the need for external power sources. This makes them highly suitable for applications in environmental monitoring, industrial automation, and smart grid systems. Overall, the versatility and efficiency of piezoelectric materials make them indispensable in modern technology, driving advancements in various sectors and contributing to the development of sustainable and innovative solutions [1-3].

Piezoelectric sensors have shown great potential in generating electricity from mechanical vibrations, making them ideally suited to integration in infrastructures such as roads and pavements. These sensors can convert the mechanical energy produced by the movement of vehicles and pedestrians into electrical energy. This innovative application has been demonstrated in several projects, including those in Toulouse, France, where piezoelectric sensors embedded in road surfaces capture the vibrations caused by traffic [4]. The harvested energy can be used

to power streetlights, traffic signals, and other roadside infrastructure, contributing to more sustainable urban environments. In Japan, similar technology has been applied to railway systems. Piezoelectric sensors installed along train tracks capture the vibrations generated by passing trains. This energy recovery system not only reduces the overall energy consumption of the rail network, but also provides a renewable energy source that can be fed back into the network or used to power auxiliary systems such as lighting and signalling. These implementations highlight the versatility and efficiency of piezoelectric materials in harnessing otherwise wasted mechanical energy. Moreover, the integration of piezoelectric sensors into transportation infrastructure offers additional benefits. For instance, the data collected from these sensors can be used for monitoring the health and performance of the infrastructure. By analyzing the vibration patterns, it is possible to detect early signs of wear and tear or structural issues, enabling timely maintenance and reducing the risk of failures. This dual functionality of energy harvesting and structural health monitoring makes piezoelectric sensors a valuable addition to smart city initiatives and intelligent transportation systems. Overall, the use of piezoelectric sensors in roads, pavements, and railway systems exemplifies how advanced materials can contribute to energy efficiency and sustainability in urban environments. These projects in Toulouse and Japan serve as pioneering examples of how piezoelectric technology can be effectively utilized to harness renewable energy from everyday activities, paving the way for broader adoption in various infrastructure applications [4, 5]. The benefits of piezoelectric materials encompass their durability and ability to operate in harsh environments, but they also present challenges, such as conversion efficiency and production costs [6]. The positive environmental impact of using piezoelectric materials for energy recovery is notable, as it helps reduce dependence on non-renewable energy sources [7]. Additionally, piezoelectric materials can be used to actively control vibrations in structures, thereby improving the stability and lifespan of buildings and bridges [8]. Ongoing research & development in this field is exploring new material compositions and improvements in manufacturing techniques, laying the foundations for future advances [9]. In summary, piezoelectric materials offer immense potential for multiple uses, ranging from vibration energy recovery to active vibration management, while contributing to a more sustainable energy future [10]. Hamid worked on a project aimed at controlling the amplitude of oscillations in wind turbine blades for two purposes: first, to eliminate blade vibrations caused by wind attack or other sources, and second, to generate ultrasonic vibratory waves to remove accumulated ice from the wind turbine blades.

The integration of functionally graded materials (FGMs) in the manufacturing of wind turbine blades represents a significant advancement in the field of renewable energy [11]. FGMs, with their gradually varying chemical composition or microstructure, allow for the optimization of mechanical and thermal properties of the blades, thereby enhancing their strength and durability. By using FGMs, the blades can better withstand varying stress and vibrations caused by wind gusts, reducing the risk of cracks and structural failures. Additionally, FGMs can be designed to enhance aerodynamic efficiency of the blades, thereby increasing the overall efficiency of wind turbines [12]. This technology also enables the reduction of blade weight without compromising their robustness, which is crucial for large offshore wind turbines [13]. In summary, the integration of FGMs in the manufacturing of wind turbine blades offers promising prospects for improving the reliability and efficiency of wind energy systems, while contributing to more sustainable and environmentally friendly energy production. Functionally graded materials (FGMs) are innovative materials whose properties gradually change depending on the position. However, during their fabrication, porosities can occur, significantly affecting their mechanical performance. These porosities, often unavoidable, can alter stress distribution and reduce the overall strength of the material. In the medical field, for example, FGMs with porosity gradients are studied to mimic bone structure, providing implants that are more compatible with human

tissue. Controlling the size and distribution of pores remains a major challenge, but advanced techniques like additive manufacturing allow for better control of these parameters. In summary, while porosities can pose challenges, they also offer unique opportunities to enhance the applications of FGMs in various fields. Otherwise, it should be mentioned that beams generally are extensively utilized in aerospace engineering and various other industries, and specifically, variable section beams are very important structural elements in the building and structural industry because they offer various advantages for builders, including excellent stiffness and stable structure. The use of these beams allows builders to reduce material and labor costs, and provide excellent rigidity and structural stability. El Harti et al. studied the control of the dynamics of a non-uniform section beam, based on Timoshenko's theory [14]. Shabani and Cunedoglu, treated the Free vibration analysis of cracked functionally graded non-uniform beams [15]. El Harti et al. have also considered the influence of the porosity of FGM materials in [16, 17] and the influence of the thermal effect in [16-18].

The aim of this work is to actively control the dynamics of an FGM beam with a non-uniform cross-section. The law describing the variation of the FGM material is exponential (E-FGM) with the presence of porosity. The mathematical modeling is based on Euler-Bernoulli beam's theory combined with the FEM.

## 2. MATHEMATICAL MODELING

In our study, we consider the cantilever FGM beam of length  $L$ , variable width  $b(x)$  and thickness  $h$ , composed of a finite number of elements. The upper surface is made of pure ceramic and the lower surface is pure metal. In the upper and lower surface four layers of piezoelectric material are symmetrically bonded, functioning as a sensor/actuator. In this study, the beam retracts linearly ( $m=1$ ). The cross-section  $A(x)$ , width  $b(x)$  and moment of inertia  $I_y(x)$  of the beam are given by [19] as follows:

$$b(x) = b_0 \left(1 - C_b \frac{x}{L}\right)^m \tag{1}$$

$$A(x) = A_0 \left(1 - C_b \frac{x}{L}\right)^m \tag{2}$$

$$I_y(x) = I_{y0} \left(1 - C_b \frac{x}{L}\right)^m \tag{3}$$

For  $A(x)$  and  $I_y(x)$  to be positive, the width taper-ratio  $C_b$  must satisfy  $0 \leq C_b < 1$ . The width taper-ratio can be calculated by knowing the widths at the embedding and the free end of the structure as follows:

$$C_b = 1 - \frac{b}{b_0} \tag{4}$$

The values for the cross-sectional area and moment of inertia at the point where the beam is embedded can be expressed as follows:

$$\begin{aligned} A_0 &= b_0 h \\ I_{y0} &= \frac{b_0 h^3}{12} \end{aligned} \tag{5}$$

In this study, we consider exponential law that describe the variation of the FGM material with porosity along the thickness axis presented as follows [20]:

$$S(z) = S_m e^{\left( \rho \left( \frac{2z+h}{2h} \right)^k \frac{2n}{1-n} \right)} \quad (6)$$

$$\rho = \ln \frac{S_c}{S_m} \quad (7)$$

where the material's effective properties  $S(z)$  is for density  $\rho(z)$ , and Young's modulus  $E(z)$ . The subscript "m" and "c" show the metal/ceramic respectively;  $k$  ( $0 \leq k \leq \infty$ ) shows the power index, and  $n$  ( $0 \leq n \leq 1$ ) is the porosity volume fraction index. The axial and transverse displacements,  $u$  and  $w$ , of any point on the beam are given follows:

$$u(x, y, z, t) = z \frac{\partial w(x, t)}{\partial x} \quad w(x, y, z, t) = w(x, t) \quad (8)$$

Strain/stress component are given by:

$$\varepsilon_{xx} = z \frac{\partial^2 w(x, t)}{\partial x^2} \quad \sigma_{xx} = Ez \frac{\partial^2 w(x, t)}{\partial x^2} \quad (9)$$

Applying Hamilton's generalized principle yields:

$$\delta \int_{t_1}^{t_2} (u - T - W) dt = 0 \quad (10)$$

Strain energy ( $U$ ), kinetic energy ( $T$ ) and the work performed by the distributed transverse load  $f(x)$ , are given given by:

$$U = \frac{1}{2} \iiint_V \{\sigma\}^T \{\varepsilon\} dV = \frac{1}{2} \int_0^l \iint_A Ez^2 \left( \frac{\partial^2 w}{\partial x^2} \right)^2 dA dx \quad (11)$$

$$T = \frac{1}{2} \iiint_V \rho \left( \frac{\partial w}{\partial t} \right)^2 dV = \frac{1}{2} \int_0^l \iint_A \rho \left( \frac{\partial w}{\partial t} \right)^2 dA dx \quad (12)$$

$$W = \int_0^l f w dx \quad (13)$$

To derive the finite element equations for the beam element, the beam is segmented into a finite number of elements, each with an equal length  $l$ . By applying the boundary conditions for the embedded-free beam as specified in [15, 16],  $w(x, t)$  is determined as follows:

$$[W(x, t)] = [f_1(x) \ f_2(x) \ f_3(x) \ f_4(x)] \times [w_1 \ \theta_1 \ w_2 \ \theta_2]^T = [n^T][q] \quad (14)$$

$$[w'(x, t)] = [n_2^T][q] \quad (15)$$

$$[w''(x, t)] = [n_1^T][q] \quad (16)$$

$$[\dot{w}(x, t)] = [n_3^T][\dot{q}] \quad (17)$$

Where  $[n]$  is the shape function,  $[n_2]$ ,  $[n_1]$  are its first and second derivatives, and  $[q]$  is the nodal vector of two nodes.

Replacing "Eq. (11) and (12)" in the Lagrange equation:

$$\frac{d}{dt} \left[ \frac{\partial T}{\partial \dot{q}_i} \right] + \left[ \frac{\partial U}{\partial q_i} \right] = [Z_i] \quad (18)$$

Which becomes:

$$M\ddot{q} + Kq = f(t) \quad (19)$$

Where:

$$[M] = [M^{FGM}] + [M^s] + [M^a] \quad (20)$$

$$[K] = [K^{FGM}] + [K^s] + [K^a] \quad (21)$$

$[M^{FGM}]$ ,  $[M^s]$ ,  $[M^a]$  and  $[K^{FGM}]$ ,  $[K^s]$ ,  $[K^a]$  are the fundamental mass/stiffness matrices of the FGM and piezoelectric sensor/actuator elements provided by [16].

The actuator generates the control force by:

$$f_{ctr} = E_p d_{31} b \bar{z} \int n_2 dx V^a(t) \quad (22)$$

$E_p$  is the Young's modulus of the piezoelectric material. And as long as there are two piezoelectric actuators symmetrically bonded to the structure to eliminate the membrane effect, there are two control forces, represented as follows:

$$f_{ctr1} = f_{ctr2} = hV^a(t) = hu(t) \quad (23)$$

With:

$$h^T = E_a d_{31} b \bar{z} [-1 \ 0 \ 1 \ 0] \quad (24)$$

When the beam is additionally subjected to an external force  $f_{ext}$ , the resulting total force vector acting on the beam is:

$$f^t = f_{ext} + f_{ctr1} + f_{ctr2} + f_{th} \quad (25)$$

Using Rayleigh's proportional damping:

$$C = \alpha M + \beta K \quad (26)$$

In order to be interested in the first modes of vibration, we have used the generalized coordinates with transformation  $q = Tg$  : where  $T$  is the modal matrix and  $g$  is the vector of principal coordinates. The structural dynamic equation and the control equation are provided as follows:

$$M^* \ddot{g} + C^* \dot{g} + K^* g = f_{ext}^* + f_{ctr1}^* + f_{ctr2}^* \quad (27)$$

The state/space representation in MIMO (Multi-Input Multi-Output) mode is described as follows:

$$\dot{x} = Ax(t) + Bu(t) + Er(t) \quad (28)$$

$$y(t) = C^T x(t) + Du(t) \quad (29)$$

The method of LQG (Linear Quadratic Gaussian) is a method widely used by the mechanics [16-18], based on the desire to minimize performance J index of the form:

$$J(u) = \int_0^{\infty} (x^t Q x + u^t R u) dt \quad (30)$$

where  $[Q]$  and  $[R]$  are respectively, the matrices of defined semi-positive and defined positive weighting on outputs and control inputs.

### 3. RESULTS AND DISCUSSION

This study focuses on the active vibration control of a variable section functionally graded material (FGM) beam. It examines the effects of the porosity of the FG material and the variation of the taper ratio. Exponential laws are used to define the variation of the FGM material along the thickness axis. For this purpose, an FGM beam, fixed at its left end and free at the other end,

is considered. Four layers of piezoelectric materials are symmetrically bonded to the upper and lower faces of the beam, acting as sensors/actuators. The beam's dimensions are  $l \times b \times h = 0.5 \times 0.02 \times 0.0001 \text{ m}^3$ .

The following figures 1, 2 and 3, present the controlled & uncontrolled impulse responses of the exponential law, varying the width  $b$  "Figure 1", the porosity index  $n$  "Figure 2" and the power index  $k$  "Figure 3".

Figure 1 shows the impulse response, varying the value of the width  $b$  ( $b= 2; 2.4; 2.8$ ) while keeping the power index  $k=1$  and the porosity index  $n=1$ .

From Figure 1, we can see that increasing the width  $b$ , implies a reduction in vibration amplitudes.

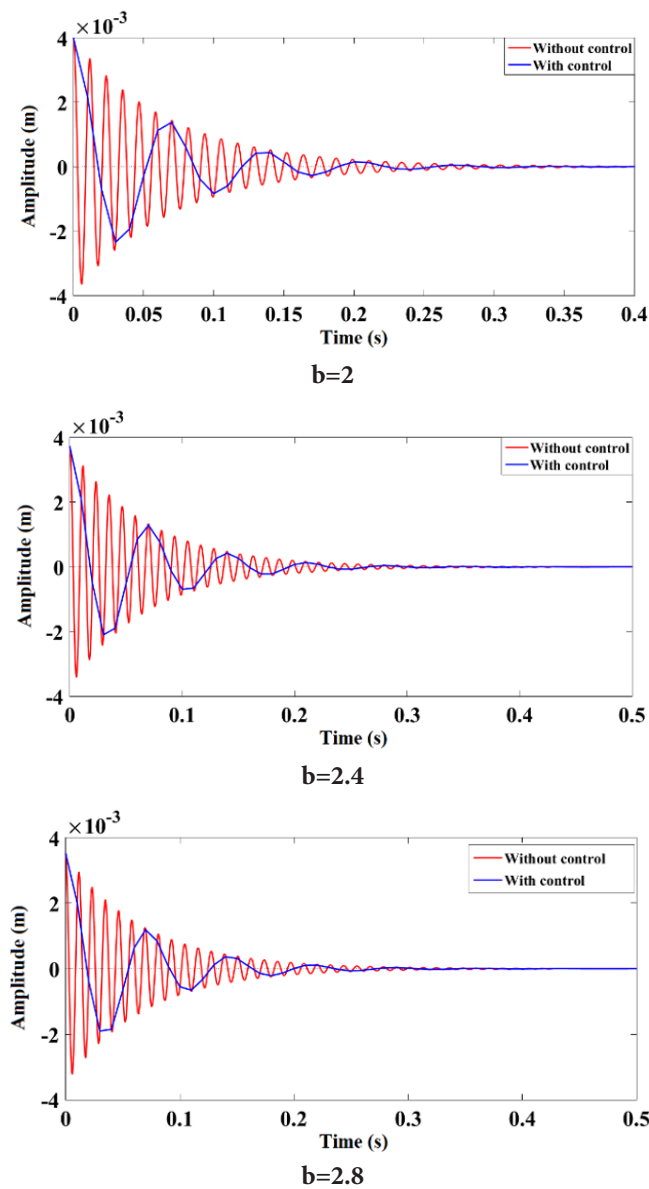


Figure 1. Impulse response of the exponential law, by varying the width  $b$ , ( $k=1; n=0; m=1$ ).

Figure 2 shows the impulse response, varying the value of the porosity index  $n$ , ( $n=0; 0.1; 0.2$ ), and keeping the power index  $k=1$  and the width  $b=2.8$ .

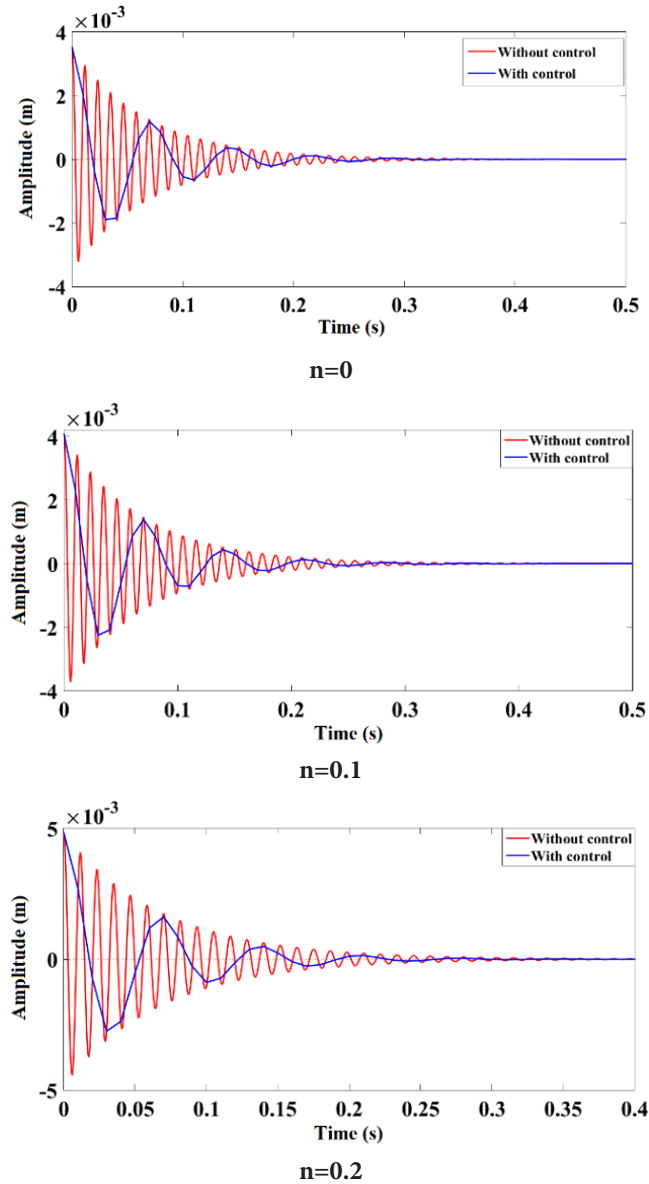
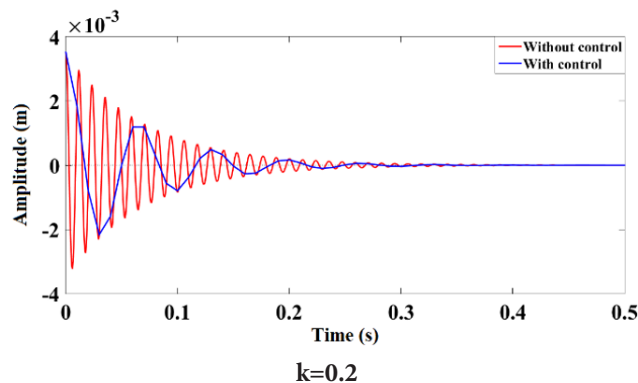


Figure 2. Impulse response of the exponential law, by varying the porosity index  $n$ , ( $k=1$ ;  $b=2.8$ ;  $m=1$ ).

From Figure 2, we can see that increasing the porosity index implies increases in vibration amplitudes. Figure 3 shows the variation in power index  $k$  ( $k=0.2$ ;  $k=1$ ;  $k=5$ ) and the comparison of controlled responses, keeping the porosity index  $n=0$  and width  $b=2.8$ . From this figure, we can see that the increase in power index implies increases in vibration amplitudes for both controlled and uncontrolled responses.





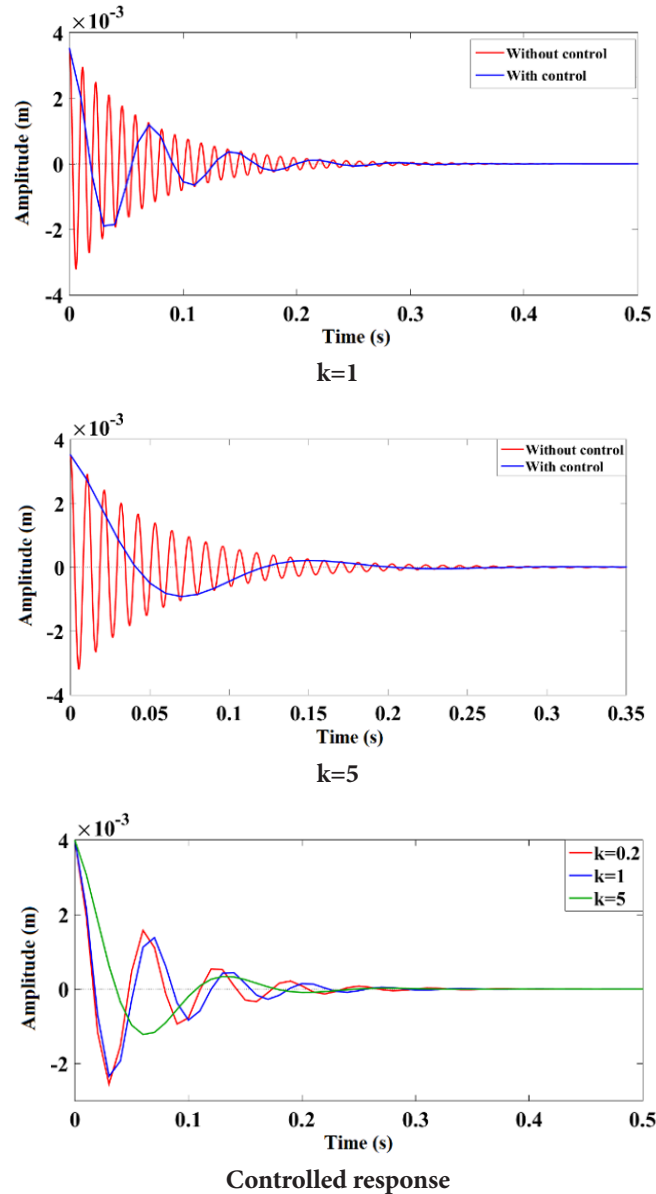


Figure 3. Impulse response of the exponential law, by varying the power index  $k$ , ( $n=0$ ;  $b=2.8$ ;  $m=1$ ).

Figures 1, 2, and 3 show that decreasing width, increasing porosity index, and increasing power index lead to increases in vibration amplitudes. It is important to note that when ( $k = 0$ ), the beam is made of pure ceramic. As the porosity index ( $k$ ) increases, the beam contains more and more metal, which negatively affects the stiffness of the beam. Additionally, the presence of porosity in the FGM material reduces the stiffness modulus, which explains the increase in vibration amplitudes.

#### 4. CONCLUSION

To validate the control method, we examined an embedded-free FGM beam with a non-uniform cross-section, partially covered by four layers of piezoelectric materials. A pulse force of 1N was applied externally. The figures below depict the beam's response to this pulse. The dynamics and vibration control outcomes were derived using the linear quadratic Gaussian (LQG) control method combined with Kalman filtering. The exponential law was utilized to describe the variation in the properties of the FGM material across its thickness. The results indicate that a reduction in width, an increase in the porosity index, and a higher power index lead to increased vibration amplitudes.

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